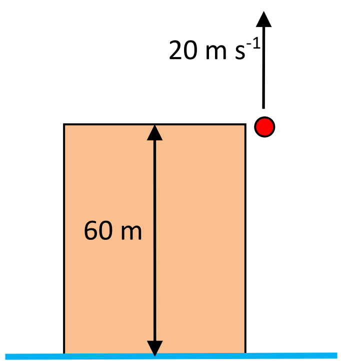
Teacher notes
Topic A

Motion under gravity

In this problem we will take $g = 10\ \text{ms}^{-2}$.

The following problem is as typical as it gets. Here are some approaches.

A ball is thrown vertically upwards with speed 20 m s^{-1} from the edge of a cliff of height 60 m. When will the ball hit the sea?

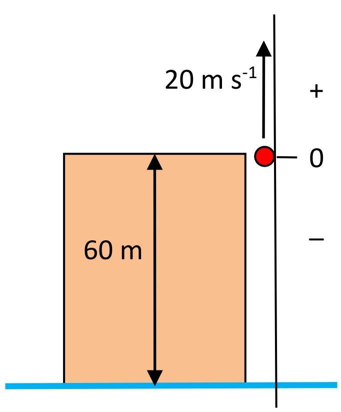


We must be very clear about the **conventions** we will use:

The first step is to draw a vertical line along which the motion takes place.

Then place the zero of the line anywhere you want. A good choice is where the motion starts.

Then decide which direction is positive and which is negative. This will determine whether your acceleration is to be taken positive or negative.



We want to find the time when the ball hits the sea. According to our conventions this happens when the **position** of the ball is – 60 m. Then we go to a formula involving displacement and time i.e. $s = ut + \frac{1}{2}at^2$. Remember *s* is **displacement** i.e. change of position i.e. $s = y - y_0$. Thus

$$y = y_0 + ut + \frac{1}{2}at^2$$

According to our conventions the initial position is $y_0 = 0$, *u* = + 20 m s^{-1} and *a* = – *g* = – 10 m s^{-2}. Thus

$$-60 = 20t - 5t^2 \text{ or } 5t^2 - 20t - 60 = 0 \text{ or } t^2 - 4t - 12 = 0.$$

This method forces you to solve a quadratic equation. The answers are *t* = – 2 s (rejected; is there a physical meaning to this answer?) and *t* = 6 s.

Can we avoid the quadratic equation? The answer is we can. We can easily find the time the ball gets to its highest point; the velocity is zero there and by our conventions:

$v = u + at$ i.e. $0 = +20 - 10t \Rightarrow t = 2\text{ s}$. The maximum height reached (from the cliff) is $y_{max} = +20\times 2 - 5\times 2^2 = 20\text{ m}$ or, using $v^2 = u^2 + 2ay$ we get $0 = 20^2 - 2\times 10\times y_{max} \Rightarrow y_{max} = 20\text{ m}$.

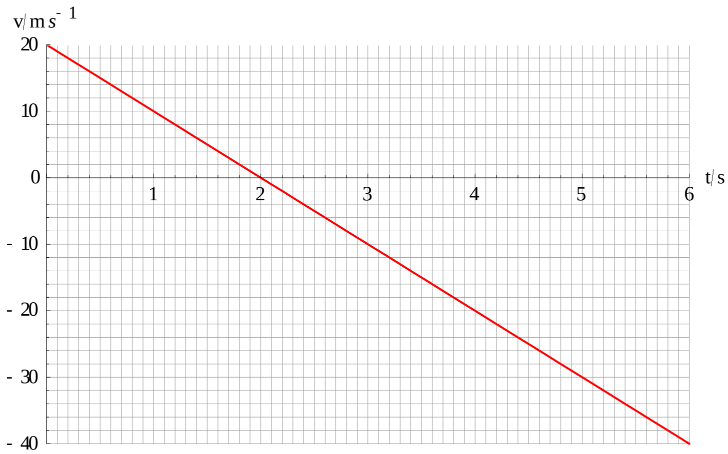
At the highest point *v* = 0 and *we take this as the new starting point*, i.e. we have changed one of our conventions. The starting point is now 20 + 60 = 80 m from the sea. Using $y = y_0 + ut + \frac{1}{2}at^2$ with $y_0 = 0$ and now *u* = 0, we get

$$-80 = -5\times t^2 \Rightarrow t = 4\text{ s}$$

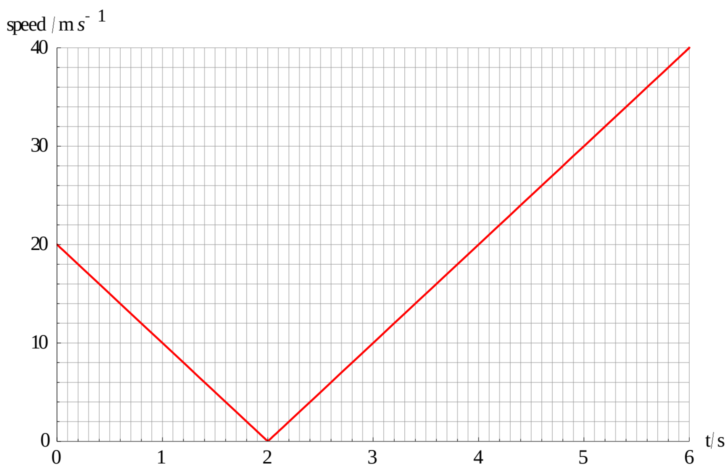
The ball took 2 s to get to the top and 4 s to fall to the sea so the total time is 2 + 4 = 6 s.

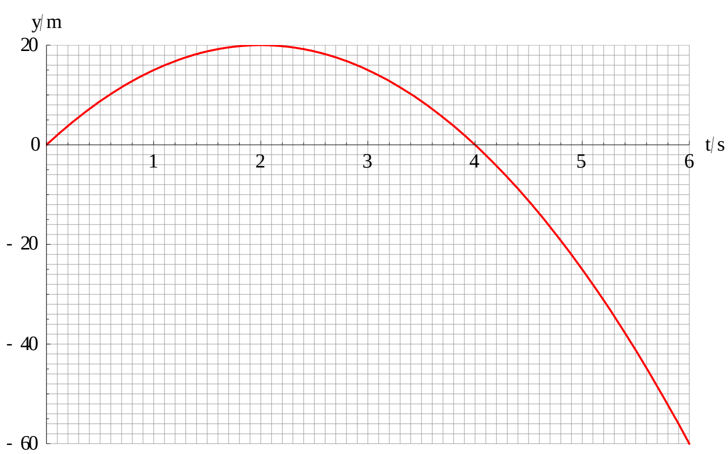
Many would change the conventions here and would take the *down direction to be positive*. In that case the relevant equation is (*a* = +*g* = + 10 m s^{-2}) $y = y_0 + ut + \frac{1}{2}at^2 = 0 + 0 + 5t^2$ and so $+80 = +5\times t^2 \Rightarrow t = 4\text{ s}$.

Graphs showing the variation with time of the velocity, speed and position of the ball are shown.



What is the gradient of this graph? What is the area under the graph from t = 0 to t = 2 s and what does it represent?





It is very instructive and it is strongly recommended to repeat this problem using different conventions. For example taking the down direction as positive and changing the position of where we put the zero on the number line. With experience, you will find that it is sometimes advantageous to change the conventions for different parts of the same problem.